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1988 J. Phys. A: Math. Gen. 21 4637

(http://iopscience.iop.org/0305-4470/21/24/017)

J. Phys. A: Math. Gen. 21 (1988) 4637-4647. Printed in the UK

Surface structure of random aggregates on the Cayley tree

Joachim Krug

Sektion Physik der Universität München, Theoretische Physik, Theresienstrasse 37, D-8000 München 2, Federal Republic of Germany

Received 19 July 1988

Abstract. Growth through ballistic aggregation and biased diffusion-limited aggregation is investigated on the Cayley tree. For a general branching ratio it is shown that the surface width of a ballistic aggregate remains finite (of order one) as the cluster mass goes to infinity. DLA with an attractive bias is treated approximately. For non-zero bias strength the surface width is asymptotically finite. In the limit of isotropic diffusion (zero bias) a roughening transition occurs which can be described in terms of a single diverging length scale.

1. Introduction

The surface properties of random aggregates have been the subject of several recent studies [1-9]. The simplest models for aggregation are those where the probability of adding a new particle depends only on the *local* environment of the growth site. Popular examples are the Eden model [1-6] and ballistic aggregation [2, 7, 10]. In the Eden model, the new particle is added with equal probability to any empty perimeter site of the cluster. In ballistic aggregation, the particle moves along a straight line towards the cluster and sticks upon contact. While the bulk of such aggregates is compact, the surface shows interesting scaling properties. In analogy with critical phenomena, one expects the large-scale structure to depend only on some general features of the growth rule. Thus one attempts a classification of growth processes with regard to their scaling properties on large length and time scales.

An important step in this direction was taken by Kardar, Parisi and Zhang, who described the dynamics of a growing surface by a non-linear Langevin equation (the KPZ equation) [9]. In two dimensions (or, equivalently, for a one-dimensional surface) the predictions of the KPZ equation agree well with numerical simulations of both the Eden model [4, 5] and ballistic aggregation [7]. This has led to the suggestion that the two growth rules might belong to the same universality class [8]. However, the situation is much less clear in higher dimensions ($d \geqslant 3$). On the one hand, accurate numerical data become increasingly difficult to obtain [6, 7]. On the other hand, the scaling properties of the KPZ equation are well understood only in two dimensions [9, 11-13].

In view of this situation, it is natural to study local growth processes on the Cayley tree, which corresponds in some sense to the limit of infinite spatial dimensionality. It has been established both for the Eden model [14] and for ballistic aggregation [15] on the Cayley tree that the radius R of a cluster grows with its mass, N, as $R \propto \ln(N)$, as one would expect for a compact object in $d = \infty$. However, the surface properties of the two models turn out to be completely different. For the Eden model [14], the width of the surface region was shown to diverge as $R^{1/2}$. In contrast, as I will demonstrate below in § 3, the surface width is asymptotically *finite* (of order one) for ballistic aggregation. A possible interpretation of this result in terms of the universality picture is proposed in § 5.

Due to the absence of screening, the Eden model on the Cayley tree is in fact equivalent to diffusion-limited aggregation (DLA) [14]. By imposing a spatial bias in addition to the diffusive motion of the aggregating particle, one obtains a model which interpolates between DLA and ballistic aggregation [16-19]. On the Cayley tree one expects, then, a transition in the surface structure as one passes from strongly biased diffusion (ballistic aggregation) to isotropic diffusion (Eden growth). Within a certain approximation, which will be explained in § 2, it is shown in § 4 that this 'roughening transition' occurs at zero bias strength. This is similar to the observed transition from compact to fractal bulk structure for biased DLA in two dimensions [16, 17]. The crossover to isotropic diffusion can be described in terms of a single diverging length scale perpendicular to the surface.

2. Cluster growth on the Cayley tree

In this section the models of interest are introduced and the growth equations are set up. The presentation follows closely that of Vannimenus *et al* [14]. We consider a tree of constant branching ratio $K \geqslant 1$. The levels are numbered by $m = 0, 1, 2, 3, \ldots$, and there are K^m sites on level m. The total number of sites up to level m is therefore

$$V(m) = (K^m - 1)/(K - 1). \tag{2.1}$$

Equivalently, a regular dense packing of N particles fills the tree up to the level

$$m_0(N) = \ln(N)/\ln(K) + \ln(K - 1)/\ln(K) \tag{2.2}$$

where $m_0(N)$ is the radius of a maximally compact cluster of mass N. It will be useful as a reference length when describing the structure of random aggregates below in §§ 3 and 4.

The growth is initiated by placing a seed particle at the root ($m = 0$). The next particle is then introduced far away from the root. It performs a random walk on the tree and becomes part of the cluster when it reaches a growth site, i.e. an empty site which is the nearest neighbour of an occupied site. The diffusing particle may also escape to infinity, in which case it is discarded and a new particle is launched. To allow for biased diffusion, we let the particle go towards the root with probability α, and enter one of the K outgoing branches with probability $(1 - \alpha)/K$. For $\alpha < 1/(1 + K)$ the particle experiences a *repulsive* bias. The cluster radius was found numerically to grow linearly, $R \propto N$, in this case [14]. In the present paper I consider only the range of *attractive* bias, $1/(1 + K) \leqslant \alpha \leqslant 1$. The two limiting cases, $\alpha = 1$ and $\alpha = 1/(1 + K)$, correspond to ballistic aggregation and isotropic DLA, respectively.

For a given cluster of N particles, we denote by $a_m(N)$ the number of occupied sites on level m, and by

$$b_m(N) = Ka_{m-1}(N) - a_m(N) \tag{2.3}$$

the number of growth sites on level m. Then obviously

$$\sum_{m=0}^{\infty} a_m(N) = N \tag{2.4}$$

and from (2.3)

$$\sum_{m=1}^{\infty} b_m(N) = (K-1)N + 1. \tag{2.5}$$

We will mostly be concerned with the density of growth sites on level m:

$$\rho_m(N) = K^{-m} b_m(N). \tag{2.6}$$

Using (2.3) and $a_0 = 1$ one finds that the $\rho_m(N)$ are normalised:

$$\sum_{m=1}^{\infty} \rho_m(N) = 1. \tag{2.7}$$

The dynamics of growth is determined by the probability $p_m(N)$ for the $(N+1)$th particle to be captured at level m. It can be shown that [14]

$$p_m(N) = (1/Z) q^{-m} b_m(N) \tag{2.8}$$

with

$$q = \begin{cases} K\alpha/(1-\alpha) & \text{for } \alpha < \frac{1}{2} \\ K & \text{for } \alpha \geq \frac{1}{2} \end{cases} \tag{2.9}$$

and

$$Z = \sum_{m=1}^{\infty} q^{-m} b_m(N). \tag{2.10}$$

Note that (2.9) implies that DLA with strongly biased diffusion ($\alpha > \frac{1}{2}$) is equivalent to ballistic aggregation ($\alpha = 1$). If $\alpha > \frac{1}{2}$, the diffusing particle is certain to join the cluster. It will thus eventually stick at the root of the subtree of vacant sites into which it was launched, just as in ballistic aggregation.

We are interested in quantities averaged over all realisations of the process, such as the average occupation numbers $\langle a_m(N) \rangle$. They evolve according to

$$\langle a_m(N+1) \rangle - \langle a_m(N) \rangle = q^{-m} \langle Z^{-1} b_m(N) \rangle \tag{2.11}$$

with initial conditions

$$\langle a_m(N=1) \rangle = \delta_{m0}. \tag{2.12}$$

Through Z the growth probability $p_m(N)$ is a non-linear function of the $a_m(N)$. Therefore the right-hand side of (2.11) cannot in general be expressed in terms of average occupation numbers. This is possible only in the limiting cases $q = K$ (strongly biased diffusion or ballistic aggregation) and $q = 1$ (isotropic diffusion). For $q = K$, $p_m(N) = \rho_m(N)$ and $Z = 1$ by (2.7). Then (2.11) reduces to a *linear* recursion relation, which is in terms of $\rho_m(N)$

$$\langle \rho_m(N+1) \rangle - \langle \rho_m(N) \rangle = K^{-(m-1)} \langle \rho_{m-1}(N) \rangle - K^{-m} \langle \rho_m(N) \rangle. \tag{2.13}$$

Similarly, for $q = 1$, $p_m(N) = (1/Z) b_m(N)$, which means that all growth sites have *equal* probability of being occupied. The growth rule thus reduces to the Eden model and $Z = (K-1)N + 1$, the total number of growth sites (2.5). This case was studied in detail by Vannimenus *et al* [14]. The growth equation (2.13) will be solved in the following section.

To make progress in the case of general q, the most obvious approximation [20] consists of replacing Z in (2.11) by its average $\langle Z\rangle(N)$. This then yields a non-linear recursion:

$$\langle\rho_m(N+1)\rangle-\langle\rho_m(N)\rangle=\langle Z\rangle(N)^{-1}(q^{-(m-1)}\langle\rho_{m-1}(N)\rangle-q^{-m}\langle\rho_m(N)\rangle). \tag{2.14}$$

It is instructive to derive this equation from a somewhat different point of view. To this end we redefine the growth process as follows. Instead of adding particles one by one, suppose that a growth site at level m is occupied independently with probability $q^{-m}\,\mathrm{d}s$ in the time interval $\mathrm{d}s$. Then the growth site distribution evolves according to

$$\frac{\mathrm{d}}{\mathrm{d}s}\langle\!\langle\rho_m(s)\rangle\!\rangle=q^{-(m-1)}\langle\!\langle\rho_{m-1}(s)\rangle\!\rangle-q^{-m}\langle\!\langle\rho_m(s)\rangle\!\rangle \tag{2.15}$$

where the double brackets indicate that we are now dealing with a different ensemble of realisations: while (2.11) and (2.13) hold for clusters of fixed mass N, the cluster mass at fixed time s is a random variable. The number of particles deposited in time $\mathrm{d}s$ equals $Z\,\mathrm{d}s$. As will be shown below, in (4.4), the average $\langle\!\langle Z\rangle\!\rangle(s)$ grows as a power of s for $q<K$. According to the law of large numbers, one expects then the fluctuations in the cluster mass N (or equivalently in Z) to become small asymptotically. Neglecting these fluctuations altogether, the two timescales N and s are simply related by a change of variables

$$\mathrm{d}N/\mathrm{d}s=\langle\!\langle Z\rangle\!\rangle(s) \tag{2.16}$$

and (2.15) becomes equivalent to (2.14). We see that the approximation effectively amounts to replacing a fluctuating timescale by its average, with the rationale that the relative fluctuations become negligible as the number of events (which equals the number of particles deposited per unit time) increases. While it seems difficult to give a rigorous justification for this approach, the results derived from it in § 4 are both consistent and plausible. As (2.15) is equivalent to (2.13) with K replaced by q, the solution for the ballistic case ($q=K$) obtained in § 3 carries over directly to the general situation.

3. Ballistic aggregation

We want to solve (2.13) with the boundary condition $\langle\rho_0(N)\rangle=0$ and initial conditions $\langle\rho_m(1)\rangle=\delta_{m1}$. As we will be dealing exclusively with ensemble averages, the brackets are dropped in the following. It is convenient to introduce a continuous time variable $t\simeq N-1$ and rewrite (2.13) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_m(t)=K^{-(m-1)}\rho_{m-1}(t)-K^{-m}\rho_m(t). \tag{3.1}$$

For $m=1$ this implies

$$\rho_1(t)=\exp(-t/K). \tag{3.2}$$

Introducing the Laplace transform $\mu_m(z)$ of $\rho_m(t)$, one obtains the recursion

$$\mu_m(z)=\frac{K^{-(m-1)}}{(z+K^{-m})}\mu_{m-1}(z) \tag{3.3}$$

for $m \geqslant 2$. Using the boundary condition (3.2) for $m = 1$, this yields

$$\mu_m(z) = K^{-m(m-1)/2} \prod_{n=1}^{m} (z + K^{-n})^{-1}. \tag{3.4}$$

The inverse transform of (3.4) is

$$\rho_m(t) = \sum_{j=0}^{m-1} (-1)^j K^{-j(j-1)/2} C_{m-j-1}\, C_j \exp(-\tau K^j) \tag{3.5}$$

where $\tau = t/K^m$ and

$$C_j = \prod_{i=1}^{j} (1 - K^{-i})^{-1}. \tag{3.6}$$

As the terms in the sum (3.5) decreases rapidly with increasing j, the upper limit of summation may be replaced by infinity for large m. Likewise the factors C_{m-j-1} may then be set equal to

$$C_\infty = \lim_{n\to\infty} C_n. \tag{3.7}$$

Thus asymptotically $\rho_m(t)$ depends on m only through the scaled time τ, i.e.

$$\rho_m(t) \simeq F_K(t/K^m) \tag{3.8}$$

with

$$F_K(\tau) = C_\infty \sum_{j=0}^{\infty} (-1)^j K^{-j(j-1)/2} C_j \exp(-\tau K^j). \tag{3.9}$$

The moments of F_K can be computed from the Laplace transform (3.4) by evaluating derivatives with respect to z at $z = 0$. One finds

$$\int_0^\infty \mathrm{d}\tau\, F_K(\tau) = 1 \tag{3.10a}$$

$$\int_0^\infty \mathrm{d}\tau\, \tau F_K(\tau) = K/(K-1) \tag{3.10b}$$

$$\int_0^\infty \mathrm{d}\tau\, \tau^2 F_K(\tau) - \left(\int_0^\infty \mathrm{d}\tau\, \tau F_K(\tau)\right)^2 = K^2/(K^2-1). \tag{3.10c}$$

A plot of $F_K(\tau)$ for $K = 2$ is shown in figure 1.

Equation (3.8) implies that multiplying the cluster mass by K, i.e. adding one monolayer of particles, simply shifts the growth site distribution by one lattice constant. It follows that the first moment

$$R(t) = \langle m \rangle_t = \sum_{m=1}^{\infty} m\rho_m(t) \tag{3.11}$$

grows as $\ln(t)/\ln(K)$. $R(t)$ is the average position of the active zone in the sense of Rácz and Plischke [1]. Up to a constant, it is equal to the reference radius $m_0(t)$ given by (2.2):

$$R(t) \simeq m_0(t) - r_0(K). \tag{3.12}$$

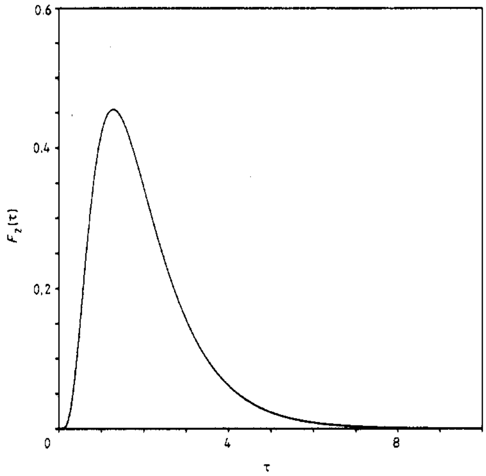

Figure 1. Scaling function $F_K(\tau)$ for $K=2$. The infinite sum (3.9) was truncated at $j=5$. The truncation error is approximately 2^{-10}.

This was already noted in [14, 15]. Furthermore, the width of the active zone is asymptotically constant:

$$\sigma(t)^2 = \langle m^2 \rangle_t - \langle m \rangle_t^2 \simeq \sigma_0(K)^2. \tag{3.13}$$

The constants $r_0(K)$ and $\sigma_0(K)$ could in principle be derived from the scaling function (3.9). It is, however, more convenient to determine them through a consistency argument. In figure 2 the asymptotic profile (3.8) is compared with a direct numerical solution of (2.13). It is clear from the picture that $\rho_m(t)$ is well approximated by a Gaussian

$$\rho_m(t) \simeq (2\pi\sigma_0^2)^{-1/2} \exp[-(m-R(t))^2/2\sigma_0^2] \tag{3.14}$$

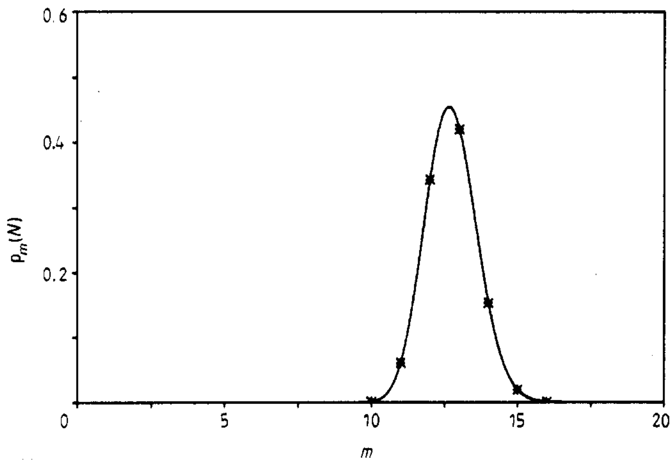

Figure 2. Growth site distribution $\rho_m(N)$ as a function of m for branching ratio $K=2$ and $N=2^{13}$. The full curve represents the asymptotic surface profile (3.8); the stars were obtained by solving numerically the difference equation (2.13).

for large t. We use (3.14) to enforce the normalisation condition (2.5) for the total number of growth sites. Replacing the sum over m by an integral, one finds that r_0 and σ_0 have to be related by

$$r_0(K)=\sigma_0(K)^2\ln(K)/2. \tag{3.15}$$

A second relation between r_0 and σ_0 is obtained by inserting the ansatz (3.14) into (3.1) and computing the first moment directly from the growth equation. Consistency then requires that

$$R(t)=\ln(t)/\ln(K)+\sigma_0^2\ln(K)/2+\ln(\ln(K))/\ln(K). \tag{3.16}$$

Together with (3.12) and (3.15) this yields

$$\sigma_0(K)^2=[\ln(K-1)-\ln(\ln(K))]/\ln(K)^2. \tag{3.17}$$

Both (3.17) and (3.15) have been checked by solving (2.13) numerically for various values of K. The agreement is excellent, which shows that the Gaussian approximation (3.14) is fully justified.

Note that $r_0>0$ if $\sigma_0>0$. Due to the roughening of the surface, the active zone lags behind the radius of a maximally compact cluster, $m_0(t)$. This shows that $R(t)$ is the radius of a dense core region of the cluster, which may contain only a small portion of the cluster mass if σ_0 is large [14]. Complementary to the inner radius $R(t)$, the outer cluster radius $\tilde{R}(t)$ can be defined as the first moment of the *number* (rather than the density) of growth sites at level m:

$$\tilde{R}(t)=[t(K-1)]^{-1}\sum_{m=1}^{\infty} mb_m(t). \tag{3.18}$$

Using again the Gaussian approximation (3.14) one finds

$$\tilde{R}(t)\simeq m_0(t)+r_0(K)=R(t)+2r_0(K) \tag{3.19}$$

which is now *larger* than m_0. The distance $\tilde{R}-R=2r_0$ is another measure for the width of the surface region. In the following I will, however, mostly use the moments (3.11) and (3.13) to characterise the cluster surface.

Before ending this section, let me briefly discuss the bearing of the present results on a related model for directed DLA (DDLA) on the Cayley tree, introduced by Bradley and Strenski [15]. In their version of DDLA, particles are injected at the *root* of the tree and execute a directed random walk, choosing randomly at each step one of the K outgoing branches. A particle joins the cluster when it reaches an empty site next to an occupied one. The model is well defined only on a *finite* tree. Once the root site is filled, the cluster is complete. The basic quantity of interest is the probability $W_L(N)$ that the root of a tree with L levels is filled by the Nth particle. Surprisingly, Bradley and Strenski found that $W_L(N)$ has a simple meaning also in the context of ballistic aggregation: it gives the probability that the ballistic aggregate *first* reaches level L when the Nth particle is added to it. In the notation of § 2

$$W_L(N)=p_L(N)\prod_{n=1}^{N-1}(1-p_L(n)). \tag{3.20}$$

Since $p_m(N)=\rho_m(N)$ for ballistic aggregation, we may apply the results for the growth site distribution to evaluate (3.20). We also make use of the fact that $W_L(N)$ is concentrated at the very tip of the cluster, where $\rho_L(N)$ is small and $L\gg R(N)$. With these simplifications, (3.20) can be transformed into

$$\ln(W_L(N))\simeq\ln(\rho_L(N))-N\rho_L(N)/2. \tag{3.21}$$

Inserting for ρ_L the Gaussian approximation (3.14), one finds that $W_L(N)$ is concentrated around the level number

$$L_0(N) \simeq \ln(N)/\ln(K) + \sigma_0(K)(2\ln(N))^{1/2} \tag{3.22}$$

which is indeed far ahead of the cluster radius $R(N)$. From (3.22) the average density (which is the number of particles divided by the number of sites) of a completed DDLA cluster of L levels can be estimated. The density $\bar{n}(L)$ decays as

$$\bar{n}(L) \propto \exp(-aL^{1/2}) \tag{3.23}$$

where a is some constant of order one. The decay is slower than geometric, as conjectured by Bradley and Strenski, but faster than a power law. I have checked that (3.23) provides a good fit to the numerical data presented in [15].

4. Aggregation through biased diffusion

For general $q > 1$, the growth probability (2.8) decays exponentially on a length scale

$$\xi(q) = 1/\ln(q) \tag{4.1}$$

on the lattice of level numbers, where ξ can be thought of as a correlation length perpendicular to the cluster surface. Growth sites in the interior of the cluster are strongly favoured, if their distance to the surface exceeds ξ. Consequently, the surface region cannot widen indefinitely. Just as for ballistic aggregation, we expect then the growth site distribution to behave asymptotically as a travelling wave:

$$\rho_m(N) \simeq G(u\ln(N) - m) \tag{4.2}$$

where the propagation speed u and the shape function G are to be determined. The correlation length diverges at $q = 1$, corresponding to the anticipated roughening of the cluster surface ($\sigma \propto R^{1/2}$) in the limit of isotropic diffusion (Eden growth). In the following I will verify this picture for the approximate growth equation (2.14).

From the results of the previous section we infer that the asymptotic solution to (2.15) is (all brackets are omitted)

$$\rho_m(s) \simeq F_q(s/q^m). \tag{4.3}$$

Inserting this into (2.10) the average of Z can be calculated:

$$Z(s) \simeq C(q, K)s^{1/\nu - 1} \tag{4.4}$$

with $\nu = \ln(q)/\ln(K)$ and C is some constant. From (2.16) we obtain the relation between the timescales s and $t \simeq N - 1$:

$$s = (t/\nu C)^{\nu}. \tag{4.5}$$

The solution (4.3) can then be written in terms of t:

$$\rho_m(t) \simeq G_q(\ln(t)/\ln(K) - \ln(\nu C)/\ln(K) - m) \tag{4.6}$$

where

$$G_q(x) = F_q(q^x). \tag{4.7}$$

This confirms the form (4.2) and shows that $u = 1/\ln(K)$ independent of q. As for ballistic aggregation ($q = K$), the cluster radius (3.11) differs from the reference length $m_0(t)$ only by a constant:

$$R(t) \simeq m_0(t) - r_0(q) - \ln(\nu C)/\ln(K) \equiv m_0(t) - \tilde{r}_0(q, K). \tag{4.8}$$

The surface width (3.13) is given by (3.17) with K replaced by q. In the isotropic limit $q \to 1$ it diverges as

$$\sigma_0(q)^2 \simeq 1/(2\ln(q)) = \xi(q)/2. \tag{4.9}$$

The correction to the radius also diverges:

$$\tilde{r}_0(q, K) \simeq \ln(K)\xi(q)/4 \tag{4.10}$$

for $q \to 1$. This is inconsistent with the results for the Eden model [14], $q = 1$; the correction $\tilde{r}_0$ then becomes proportional to the radius R, and σ grows as $R^{1/2}$. The asymptotic behaviour of $\tilde{r}_0$ and σ_0 for $q \to 1$ has been checked numerically by solving the approximate growth equation (2.14).

The correlation length $\xi(q)$ can also be used to describe the temporal crossover of the surface width. For $\ln(t) \ll \xi(q)$, the width grows proportional to $\ln(t)^{1/2}$, as in the isotropic case. For $\ln(t) \gg \xi(q)$, it saturates at $\sigma_0(q)$. This is summarised in the scaling form

$$\sigma(t)^2 = \sigma_i(t)^2 f(\xi(q)/\sigma_i(t)^2) \tag{4.11}$$

where $\sigma_i(t) \propto \ln(t)^{1/2}$ is the surface width of the isotropic (Eden) model. The scaling function satisfies $f(x \to \infty) = 1$ by construction. Moreover, it follows from (4.9) that $f(x) \simeq x/2$ for small x. Figure 3 shows some numerical data for the surface width, scaled according to (4.11). The scaling form is seen to hold over a large range in q.

Finally I want to discuss the crossover of the asymptotic cluster density, defined by

$$D(q, K) = \lim_{N \to \infty} (N/V(\tilde{R}(N))) \tag{4.12}$$

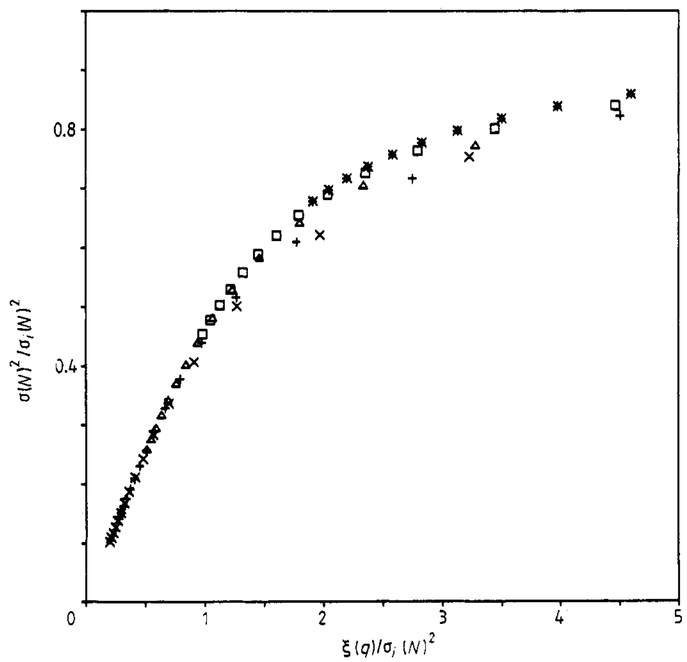

Figure 3. Numerical data for the surface width obtained from the approximate growth equation (2.14) for $K = 2$ and various values of q. The ratio $\sigma(N)^2/\sigma_i(N)^2$ is plotted as a function of $\xi(q)/\sigma_i(N)^2$. The scaling form (4.11) requires the data to collapse onto a single curve. $\times$, $q = 1.60$; $+$, $q = 1.40$; $\triangle$, $q = 1.20$; $\square$, $q = 1.10$; $*$, $q = 1.05$.

with the outer cluster radius $\tilde{R}$ given by (3.18), and the corresponding volume (equivalent to the number of sites) by (2.1). From (3.19) and (4.10) one obtains

$$D(q, K) \simeq K^{-\ln(K)\xi(q)/4}. \tag{4.13}$$

The density vanishes exponentially as $\xi \to \infty$. On a d-dimensional lattice, one would expect the density to scale as ξ^{-d}, which is consistent with (4.13) for $d = \infty$. At the critical point $q = 1$, the cluster density decays algebraically with N. Using the results of [14] one finds that $D \propto N^{-\gamma}$ with

$$\gamma = K \ln(K)/(K-1) - 1. \tag{4.14}$$

In summary, it has been shown in this section that the crossover from ballistic aggregation to isotropic DLA can be described in terms of a single correlation length $\xi(q)$. The characterisation of the isotropic limit $q = 1$, $\xi = \infty$ as a critical point for the surface structure provides a natural explanation for the delicate surface properties found in [14]. Such critical behaviour is indeed to be expected, if, as conjectured in [14], $q = 1$ represents the borderline between compact ($R \propto \ln(N)$) and linear ($R \propto N$) cluster growth. While the simplicity of the results could depend on the approximation (2.14), the general picture is expected to remain valid also for the full growth dynamics (2.11).

5. Discussion

The main conclusion of this paper is that the surface of a ballistic aggregate on the Cayley tree is smooth. This is in marked contrast to the Eden model on the same lattice. How can this result be reconciled with the idea that both processes are described by the KPZ equation? For spatial dimension $d > 3$, the KPZ equation has a weak coupling regime, corresponding to a smooth surface, and a strong coupling regime where the surface presumably is rough [11, 21]. Recent work on the related problem of directed polymers with disorder suggests that both regimes persist even in the limit of infinite dimension [22]. It is therefore quite conceivable that ballistic aggregation on the Cayley tree belongs to the weak coupling regime, whereas the Eden model corresponds to strong coupling. If this picture proves correct, the difference between the two growth rules may show already on a finite-dimensional lattice with $d > 3$.

Acknowledgments

I would like to thank Udo Seifert for some very helpful suggestions, and Herbert Spohn for a careful reading of the manuscript. This work was supported in part by Deutsche Forschungsgemeinschaft.

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